Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input: \int \frac{(z - Ru)}{(R^2 + z^2 - 2Rzu)^{(3/2)}}du

STEP 1

Take the integral:

$$\int \frac{z - R\,u}{\left(R^2 - 2\,R\,u\,z + z^2\right)^{3/2}}\,du$$

STEP 2

For the integrand $\frac{z-R\,u}{\left(R^2-2\,R\,u\,z+z^2\right)^{3/2}}$, substitute $s = \sqrt{R^2 - 2\,R\,u\,z + z^2}$ and $d\,s = -\frac{R\,z}{\sqrt{R^2-2\,R\,u\,z+z^2}}\,d\,u$:

$$= -\frac{1}{R\,z}\int \frac{-R^2 + s^2 + z^2}{2\,s^2\,z}\,d\,s$$

STEP 3

Factor out constants:

$$= -\frac{1}{2\,R\,z^2}\int \frac{-R^2 + s^2 + z^2}{s^2}\,d\,s$$

STEP 4

For the integrand $\frac{-R^2+s^2+z^2}{s^2}$, expand out the fraction:

$$= -\frac{1}{2\,R\,z^2}\int -\frac{R^2}{s^2} + \frac{z^2}{s^2} + 1\;\; d\,s$$

STEP 5

Integrate the sum term by term and factor out constants:

$$= \left(\frac{R}{2\,z^2} - \frac{1}{2\,R}\right)\int \frac{1}{s^2}\,d\,s - \frac{1}{2\,R\,z^2}\int 1\,d\,s$$

STEP 6

The integral of $\frac{1}{s^2}$ is $-\frac{1}{s}$:

$$= -\frac{\frac{R}{2\,z^2} - \frac{1}{2\,R}}{s} - \frac{1}{2\,R\,z^2}\int 1\,d\,s$$

STEP 7

The integral of 1 is s:

$$= -\frac{s}{2 R z^2} - \frac{\frac{R}{2 z^2} - \frac{1}{2 R}}{s} + \text{constant}$$

STEP 8

Substitute back for $s = \sqrt{R^2 - 2 R u z + z^2}$:

Answer:

$$= \frac{u z - R}{z^2 \sqrt{R^2 - 2 R u z + z^2}} + \text{constant}$$